

Models and images for understanding addition and subtraction

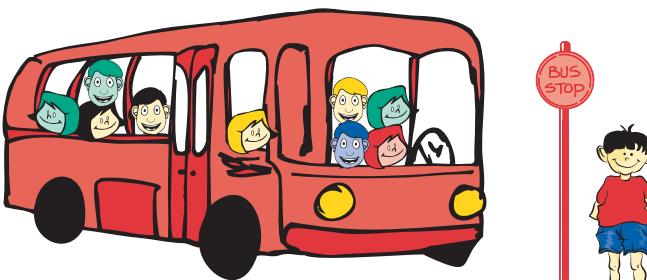
$$\begin{array}{c} \text{smiley faces} \\ \downarrow \\ \text{smiley faces} \end{array} \quad 5 = 3 + 2$$

$$3 + 2 = 5$$

$$10 = 5 + 5$$

$$10 = 1 + 9$$

$$10 = 2 + 8$$



9 and 1 more is 10

9 add 1 equals 10

9 + 1 = 10



1 less than 10 is 9

10 subtract 1 equals 9

10 - 1 = 9



$$20 = 12 + 8$$

$$\begin{array}{l} 2 + 5 = 7 \\ \text{2 count on 5} \\ \text{5 count on 2} \end{array}$$

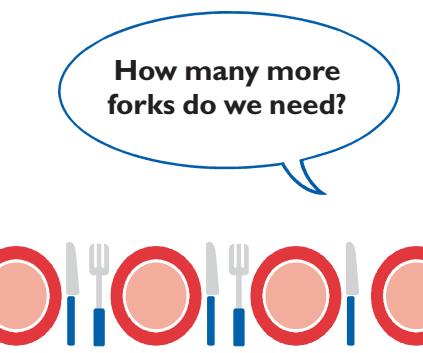
$$\begin{array}{l} 6 + 3 + 4 \\ \hline 10 + 3 \\ = 13 \end{array}$$

$$\begin{array}{l} 5 \text{ and 1 more is? } 6 \\ 5 \text{ and 2 more is? } 6, 7 \\ 5 \text{ and 3 more is? } 6, 7, 8 \\ \hline 6 \quad 7 \quad 8 \end{array}$$

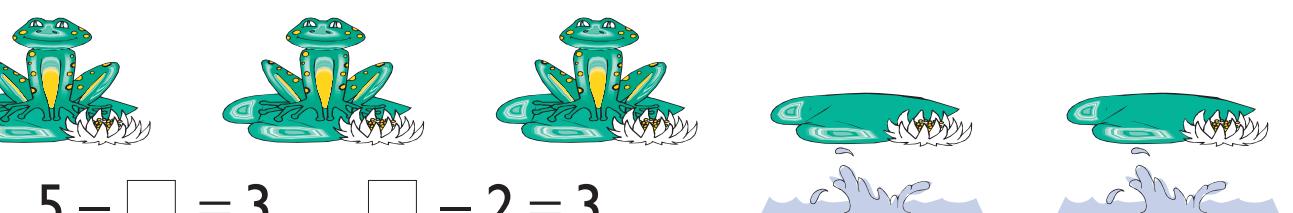
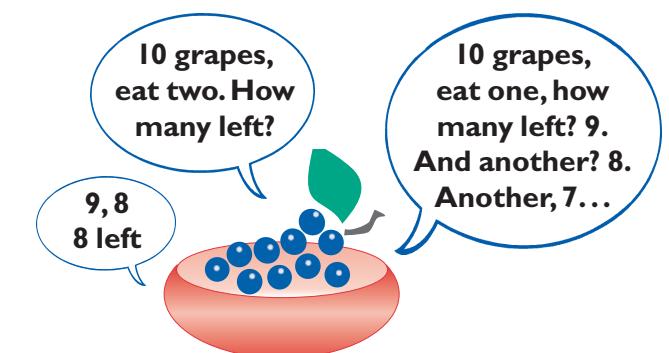
$$\begin{array}{l} \text{count on one, two or three} \\ \hline 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \end{array}$$

$$\begin{array}{l} 1 \text{ less than 8 is? } 7 \\ 2 \text{ less than 8 is? } 7, 6 \\ 3 \text{ less than 8 is? } 7, 6, 5 \\ \hline 6 \quad 7 \quad 8 \end{array}$$

$$\begin{array}{l} \text{count back one, two or three} \\ \hline 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \end{array}$$

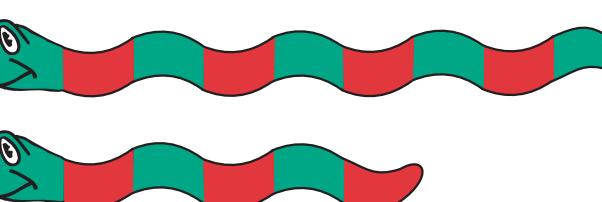


$$3 + \square = 5$$



$$\begin{array}{l} 22 - 3 \\ -1 \quad -2 \\ \hline 19 \quad 20 \quad 22 \end{array}$$

$$\begin{array}{l} 22 - 19 = 3 \\ +1 \quad +2 \\ \hline 19 \quad 20 \quad 22 \end{array}$$



Which line has **most** money?
How much **more**?

The difference is?

$$\begin{array}{l} \text{The difference} \\ \text{between 11} \\ \text{and 14 is 3.} \\ 14 - 11 = 3 \\ 11 + \square = 14 \\ \hline 0 \quad 11 \quad 14 \end{array}$$



$$\begin{array}{l} 6 \text{ and how many} \\ \text{more make 10?} \\ 6 + \square = 10 \end{array}$$

Progression

Reception

- There are three people on the bus. One more gets on. How many are there now?
- There are four children in the home corner. One leaves. How many are left?
- How many different ways can you plant five bulbs in two bowls?

Year 1

Autumn

- This domino has 3 spots on one side and 2 on the other, making 5 altogether. If I turn it around, will there still be 5?
- Start at 2, then count on 3. Start at 3, then count on 2. Will you land on the same number?
- There are 8 grapes. I eat one, how many are left? I eat another. How many are left? And if I eat another?
- There are 10 people on a bus, 2 get off, how many are left?
- This tower has 6 cubes and this one has 4 cubes. What is the difference in height?

Year 2

Autumn

- This domino has 7 spots, 3 on one side and 4 on the other. What number sentences could you write about it?
- Find three numbers with a total of 15.
- What must I add to 14 to make 16?
- There are 14 people on the top deck of a bus, and 9 below. How many people altogether?
- There are 10 grapes, you eat 4, how many are left?
- There are 20 people on a bus, 3 get off, how many people are left?
- If two towers have a difference of 3 cubes, how tall might they be?

Year 3

Autumn

- If 2 dominoes have 20 spots altogether. How many spots might be on each side of each domino?
- To work out 5 + 29 would you count on 5 or 29? Why? Does it matter which number you start at?
- You have 57p in your pocket and spend 49p, how much do you have left?
- There were 24 people on a bus. There are now only 16. How many got off? What number sentence would you write about this?

Spring

- What numbers go in the boxes? $61 + 10 = \square$; $61 + \square = 71$; $\square + 10 = 71$; $\square + \triangle = 50$
- There are 10 pegs on a coat hanger. 6 are showing, how many are hidden? What number sentences could we write?
- I think of a number. I add 10. The answer is 30. What is my number?
- What numbers go in the boxes? $86 - 50 = \square$; $26 - \square = 16$; $\square - 40 = 28$; $\square - \triangle = 40$
- You have 70p and spend 30p, what do you have left? What number sentence would you write?

Year 4

- Increase 48 by 22.
- Which three numbers could have a total of 103? Are there any others?
- What numbers go in the boxes? $91 + \square + 48 = 250$; $\square + \triangle = 1000$.

Primary National Strategy

Ref: DfES 0508-2003 G(6)

Understanding addition and subtraction

Potential difficulties

Children may:

- count forwards from one, as they are used to doing so when counting objects but cannot count from any starting number;
- count forwards confidently but lose their place and lack fluency when counting backwards, particularly when crossing the tens boundaries;
- know the answer to one more or one less, but when adding or subtracting larger numbers make mistakes as they include the first number when counting on/back, e.g. say '16, 15, 14' when subtracting 3 from 16;
- continue to rely on a combining model when adding two or more groups, having to count every object in the combined group rather than counting on from the larger group;
- not identify the number of items in a small group (subitise) and so have to count them out, consequently losing track when subtracting a small number mentally;
- use the take-away model when subtracting, e.g. $22 - 3$, counting back to 19, but continue to apply the model to calculations such as $24 - 17$ rather than use the difference model and count up from 17;
- count on or back without reference to the tens landmarks or knowledge of number facts and place value, e.g. counting on in ones from 15 when working out $26 - 15$ rather than bridging though 20;
- add and subtract by counting on or back but not recognise the inverse relationship between the operations and so not be able to derive the associated subtraction facts from addition facts or vice versa;
- count on or back in tens and ones but not combine these processes, for example when subtracting nine, subtract ten and then adjust by adding one;
- associate + with addition and – with subtraction and do calculations such as $4 + 3$ and $6 - 4$, but not be able to find missing numbers in statements such as $\square + 4 = 9$ and $\square - 4 = 5$;
- recognise what calculation to do when word problems include the words add or take away, but are less confident when other language is used such as fewer, sum, total or difference;
- when shown, use resources or models such as number lines to help with simple calculations, but not be able to apply these to unfamiliar contexts or to solving simple word problems.

Other useful models and images

Examples of progression and application in Years 4 to 6

These examples are drawn from section 6 of the *Framework for teaching mathematics from Reception to Year 6*.

Year 4

- Understand and use when appropriate the principles (but not the names) of the commutative and associative laws as they apply to addition.

$$\begin{aligned} 86 + 95 &= 95 + 86 \\ 25 + 17 + 18 &= 25 + (17 + 18) \\ &= 25 + 35 \\ &= 60 \end{aligned}$$

- Understand that subtraction is non-commutative, i.e. $5 - 7$ is not the same as $7 - 5$.

- Understand that the sum of two positive numbers is greater than either number, and that subtracting a positive number makes a number less.

- Understand that addition is the inverse of subtraction (addition reverses subtraction and vice versa) and use this to check results.

Check $625 - 87 = 538$ with
 $538 + 87 = 625$.

- Respond rapidly to oral or written questions, explaining the strategy used.

654 add 50; 63 subtract 46.
Add 15, 6, 4, 15 and 1.

- What is the sum of 26 and 39? What is the difference between 28 and 65?
Increase 48 by 22. What is 170 less than 250?

Find three numbers with a total of 103.
Find pairs of numbers with a difference of 79.

Use known number facts to rapidly answer:
 $27 + 8 = \square$; $\square + 12 = 19$;
 $27 - 19 = \square$; $43 - \square = 4$

Use a number line or mental strategies to answer:
 $76 + 58$; $\square + \triangle + \circlearrowleft = 100$;
 $136 - 78 = \square$; $\triangle - \square = 54$

Use jottings or a pencil and paper method to answer:
 $4136 + 3258 = \square$; $\triangle + \square = 1000$
 $1258 - 576 = \square$; $\square - \triangle = 682$

Year 5

- Respond rapidly to oral or written questions, explaining the strategy used.

$$\begin{aligned} 3754 \text{ add } 30; \text{subtract } 50 \text{ from } 225 \\ \square - 62 = 189; 7.6 - 5.8 = \square \end{aligned}$$

- What is the sum of 16, 64 and 153?
What is the difference between 155 and 390?
Increase 190 by 37. How many more than 952 is 1050?
570 add a number is 620. What is the number?

- Use known facts to answer:
 $\square + 62 = 189$; $7.6 + 5.8 = \square$;
 $\square - 62 = 189$; $7.6 - 5.8 = \square$

Use informal pencil and paper jottings to answer:

$$\begin{aligned} \square + 756 &= 924; \triangle + \square = 1; \\ \square - 256 &= 424; \square - \triangle = 1.2 \end{aligned}$$

Use a written method to answer:
 $14136 + 3258 + 487 = \square$;
 $141.36 - 32.58$

Use a calculator to:
find all the different totals you can make by using three of these five numbers:
8, 4008, 562, 3103, 95
Find all the differences you can make by using two of the numbers.

Year 6

- Respond rapidly to oral or written questions, explaining the strategy used.

$$\begin{aligned} \text{Add } 1200, 400, 600, 1200 \text{ and } 15. \\ \square - 2.56 = 5.38; 7.65 - 6.85 = \square \\ \text{Increase } 250 \text{ by } 420. \text{ How much less than } 4.8 \text{ is } 4.2? \\ 2.8 \text{ add a number is } 4.3. \text{ What is the number?} \\ \text{Which three numbers could have a total of } 1? \\ \text{Find pairs of numbers with a difference of } 13.5. \end{aligned}$$

- Use known facts to answer:
 $\square + 2.56 = 5.38$; $91 + \square + 48 = 250$;
 $\square - 2.56 = 5.38$; $7.65 - 6.85 = \square$

Use informal pencil and paper jottings to answer:

$$\begin{aligned} \square + 1475 &= 6924; \square + \triangle = 0.1; \\ \square - 1475 &= 2924; \square - \triangle = 0.03 \end{aligned}$$

- Use a written method to answer:
 $421.36 + 25.7 + 53.25 = \square$;
 $421.3 - 82.57 = \square$

Use a calculator to:
find all the different totals you can make by using three of these five numbers:
1.07, 0.3, 37.03, 17.73, 31.7

Find all the differences you can by using two of the numbers.

See also section 6: mental calculation strategies (pages 40–47); pencil and paper procedures (pages 48–51); check results of calculations (pages 72–73); problems involving numbers in 'real life'; money, time and measures (pages 82–89).